Modular data of non-semisimple modular categories

Liang Chang(常亮)
Nankai University

Modular tensor categories are semisimple tensor categories with nondegenerated braiding, which have many applications in low dimensional topology and topological physics. Recently, the notion of modularity is extended to non-semisimple tensor category. In this talk, we will talk about the work to extend the well-understood theory of semisimple modular categories, such as the $SL(2,Z)$ -representation and rank finiteness, to the non-semisimple case by using representations of factorizable ribbon Hopf algebras.

Combinatorial Quantization of 4d derived Chern-Simons theory and a target for ribbon 2-functors

Hank Chen(陈群皓)
BIMSA

The procedures for producing 3d TQFTs from the data of quantum group Hopf algebras are well-known. In order to lift these constructions to higher-dimensions, it is widely accepted that we need to perform a categorification. However, many unknowns and obstacles still stand in our way. As a first step towards a resolution, I will introduce a derived higher-homotopy generalization of the 3d Chern-Simons theory to 4-dimensions, and describe the higher-categorical framework in which the we can quantize it on a lattice. Based on the geometry of stratified 3-manifolds and its 2-skeleton, I will show how we can extract data from the underlying 4d action in order to equip the discrete degrees-of-freedom of 2-Chern-Simons holonomies with the structure of a Hopf cocategory.
Then, if time permits, I will discuss the representation 2-category of its quantum gauge symmetries, and demonstrate that it is ribbon tensor (and what this means). This servers as the target for a ribbon 2-functor from the Baez-Langford 2-category of 2-tangles, which completes by the 2-tangle hypothesis into a 4d "2-Chern-Simons TQFT".
This is based on my recent works https://arxiv.org/abs/2501.06486 andhttps://arxiv.org/abs/2501.08041.

A monomial basis for the Kauffman bracket skein algebra of the 4-holed disk

Haimiao Chen(陈海苗)
Beijing Technology and Business University

Fix a commutative ring R. Given an oriented surface F, its Kauffman bracket skein algebra $\mathcal{S}(F)$ is an R-algebra defined in terms of embedded links in $F \times [0,1]$, and is regarded as a quantization of the $SL(2,\mathbb{C})$-character variety of $\pi_1(F)$. It is well-known that the multicurves in F form a basis for $\mathcal{S}(F)$ as a free R-module. For F being the 4-holed disk, we present a monomial basis for $\mathcal{S}(F)$, which is more convenien in algebraic manipulations. We apply the result to compute the Kauffman bracket skein module of a knot exterior, an show a nontrivial phenomenon.

Modular extensions of $\mathrm{Rep}(G,z)$ and super orbifold theory

Chongying Dong(董崇英)
UC Santa Cruz

For any finite group G, its module category Rep(G) is a symmetric braided fusion category. The modular extensions of Rep(G) have beenstudied extensively by Muger, Drinfeld-Gelaki-Nikshych-Ostrik, Lan-Kong-Wen and others. If z is an order 2 central element, G-module category $\mathrm{Rep}(G,z)$ with a new braiding determined by z is also a symmetric braided fusion category, but the modular extensions of $\mathrm{Rep}(G,z)$ have been only known for several examples. This talk will report our recent progress in understanding the modular extensions of $\mathrm{Rep}(G,z)$ by using the super orbifold theory. This is a joint work with Richard Ng, Li Ren and Yilong Wang.

From Segal's sewing to pseudo-q-traces and back

Bin Gui(归斌)
Tsinghua University

In 1990, Zhu proved that if V is a C2 cofinite rational VOA, then the q-traces of the vertex operators for modules of V span a modular-invariant space. These q-traces have a clear geometric meaning: they are special cases of Segal's sewing construction (≈partial contractions for conformal blocks). However, if V is C2 cofinite but irrational, Miyamoto proved in 2004 that achieving modular invariance requires generalizing q-traces to pseudo-q-traces. At first glance, pseudo-q-traces do not appear to fit within Segal's sewing framework. Did Segal miss something? In this talk I will provide the answer: No. By suitably adjusting Segal's sewing, we can achieve a geometric interpretation of pseudo-q-traces. This is joint work with Hao Zhang.

Quantum Supersymmetries (II): Loewy Filtrations and Quantum de Rham Cohomology

Naihong Hu(胡乃红)
East China Normal University

We explore the indecomposable submodule structure of quantum Grassmann super-algebra $\Omega_q(m|n)$ and its truncated objects $\Omega_q(m|n,\mathbf{r})$ in the case when $q=\varepsilon$ is an ℓ-th root of unity. A net-like weave-lifting method is developed to show the indecomposability of all the homogeneous super subspaces $\Omega_q^{(s)}(m|n,\mathbf{r})$ and $\Omega_q^{(s)}(m|n)$ as $\mathcal{U}_q(\mathfrak{gl}(m|n))$-modules by defining "energy grade" to depict their "ℓ-adic" phenomenon. Their Loewy filtrations are described, the Loewy layers and dimensions are determined by combinatorial identities. The quantum super de Rham cochain short complex $(\mathcal{D}_q(m|n)^{(\bullet)},d^{\bullet})$ is constructed and proved to be acyclic (Poincar\'e Lemma), where $\mathcal{D}_q(m|n)=\Omega_q(m|n)\otimes\sqcap_q(m|n)$ and $\sqcap_q(m|n)$ is the quantum exterior super-algebra, over which we define the q-differentials such that the product structure of $\sqcap_q(m|n)$, the quantum exterior super-algebra, is well-matched everywhere. However, the truncated quantum de Rham cochain subcomplexes $(\mathcal{D}_q(m|n,\mathbf{r})^{(\bullet)},d^{\bullet})$ we mainly consider are no longer acyclic and calculating the resulting quantum super de Rham cohomologies $H^s_{DR}(\mathcal{D}_q(m|n,\mathbf{r})^{(\bullet)})$ are highly nontrivial. This is a joint work with Dr. Ge Feng and Prof. Marc Rosso.

On Symmetric Tensor Decomposition

Hualin Huang(黄华林)
Huaqiao University

In this talk, we give a brief introduction to symmetric tensor decomposition. One of the main approaches involves a natural Hopf pairing between polynomial rings and rings of differential operators. The idea is elucidated by explicit examples of binary forms.

Presentations of mapping class groups and applications to cluster algebras from surfaces

Fang Li(李方)
Zhejiang University

Let $\sum$ be Laurent phenomenon (LP) seed of rank $_n$, $\mathcal{A}(\Sigma)$, $\mathcal{U}(\Sigma)$ and $\mathcal{L}(\Sigma)$ be its corresponding Laurent phenomenon algebra, upper bound and lower bound respectively.
We prove that each seed of $\mathcal{A}(\Sigma)$ is uniquely defined by its cluster, and any two seeds of $\mathcal{A}(\Sigma)$ with $n-1$ common cluster variables are connected with each other by one step of mutation. The method in this paper also works for (totally sign-skew-symmetric) cluster algebras.
Moreover, we show that $\mathcal{U}(\Sigma)$ is invariant under seed mutations when each exchange polynomials coincides with its exchange Laurent polynomials of $\sum$.
Besides, we obtain the standard monomial bases of $\mathcal{L}(\Sigma)$.
We also prove that $\mathcal{U}(\Sigma)$ coincides with $\mathcal{L}(\Sigma)$ under certain conditions.
This is a joint work with Qiuning Du

On the partial dualization of finite-dimensional Hopf algebras: Structures, examples and gauge invariants

Kangqiao Li(李康桥)
Hangzhou Normal University

Let H be a finite-dimensional Hopf algebra with left coideal subalgebraB. There is a structure of quasi-Hopf algebra on the smash product $(H/B^+H)^* \# B$, which is called the left partially dualized quasi-Hopf algebra (or partial dual for short) of H for B . It reconstructs the dual tensor category of $\mathrm{Rep}(H)$ with respect to its left module category $\mathrm{Rep(B)}$, where $\mathrm{Rep(H)}$ and $\mathrm{Rep(B)}$ denote categories of finite-dimensional representations of respective algebras.

In this talk, we introduce the construction of partial duals, as well as:
1. Conditions when partial duals are particular;
2. Formulations of partial duals for some classical structures, including bismash products, bosonizations and quantum doubles;
3. Descriptions for gauge invariants of partial duals, such as indicators and exponent.

Matched pairs, braiding operators and the Yang-Baxter equation

Yunnan Li(黎允楠)

Guangzhou University

An effective and fruitful approach to study set-theoretic solutions of the Yang-Baxter equation is to identify and investigate the underlying algebraic structures. In this talk, I will introduce several equivalent algebraic tools, namely matched pair and braiding operator, etc, constructed by groups and also Hopf algebras, for finding set-theoretic type solutions to the Yang-Baxter equation, and then discuss some related problems being concerned.

Representation type of Hopf algebras with Chevalley property

Gongxiang Liu(刘公祥)
Nanjing University

In this talk, we try to talk about Hopf algebras with Chevalley property of finite, tame and discrete corepresentation typies. This is a joint work with Yu Jing.

New Tensor Categories from Sieving Forests

Fan Lu(陆凡)
Tsinghua University

We discovered several new examples of tensor categories from our classification results on fusion bialgebras with exchange relations, one of which is the binary tree tensor category. This classification scheme begins by transforming infinite diagrammatic consistency equations of exchange relations into a finite set of algebraic equations of degree at most 3. We then introduce a key concept, the fusion graph of a fusion bialgebra, and prove that the fusion graph for any minimal projection is a forest if and only if the fusion bialgebra has an exchange relation. For each forest fusion graph, the system of degree 3 equations reduces to linear and quadratic equations that are efficiently solvable. To deal with exponentially many forest fusion graphs in the unitary case, we propose two analytic criteria to sieve most candidates from subfactor planar algebras. We developed a computer program to compute the Grobner basis of the consistency equations for each forest fusion graph up to 6-dimension, and to sieve forest fusion graphs using our criteria without solving the equations. New examples of fusion bialgebras with exchange relations corresponding to new tensor categories were found in this process. The sieving criteria are much more efficient than directly solving the equations, demonstrating the advantages of quantum Fourier analysis.

Geometric construction of quantum Schur algebras

Li Luo(罗栗)

East China Normal University

We provide the geometric construction of a series of generalized Schur algebras of any type via Borel-Moore homologies and equivariant K-groups of generalized Steinberg varieties. As applications, we obtain a Schur algebra analogue of the local geometric Langlands correspondence of any type, provide an equivariant K-theoretic realization of quasi-split i-quantum groups of affine type AIII, and establish a geometric Howe duality for affine i-quantum groups.

On Kac-Wakimoto hypothesis

Li Ren(任丽)
Sichuan University

Motivated by the earlier work of Kac-Wakimoto on the coset constructions associated with affine vertex operator algebras, the categorial coset constructions are investigated and Kac-Wakimoto Hypothesis is proved under some mild conditions. In particular, the field identifications are obtained. These results are applied to the coset constructions in the theory of vertex operator algebras. This is a joint work with C.Dong and F.Xu.

Recent progress on (2+1)-alterfold TQFTs

Yilong Wang(王亦龙)
BIMSA

In this talk, we will give an overview of our joint program with Zhengwei Liu, Shuang Ming and Jinsong Wu on the development of the (2+1)-alterfold theory. We will demonstrate how concepts in the familiar categorical constructions of (2+1)-TQFTs are topologized and unified in the framework of alterfolds, which in particular results in a quick proof of the "RT=TV" type of result. We will then show how Morita contexts can be naturally included in the alterfold theory. Finally, if permits, we will explain how major results on the alpha-induction construction follows from intuititve alterfold calculus and how we generalize them based on such intuition.

Numerical Homological Regularities

Quanshui Wu(吴泉水)
Fudan University

Inspired by the studies in algebraic geometry and commutative algebra, Jorgensen first defined CM-regularities for graded modules over noncommutative noetherian connected graded algebras. Two fundamental commutative results are generalized to the non-commutative case: a vanishing-theorem by Mumford, and a theorem on linear resolutions and syzygies by Eisenbud and Goto. Subsequently, Jorgensen established two inequalities relating CM-regularity to Tor-regularity, which spurred numerous intriguing research efforts. For example, Romer gave a characterization that a commutative standard graded algebra is a polynomial algebra if and only if either of the two of Jorgensen's inequalities is always an equality for any finitely generated graded module. Dong and Wu generalized Romer's result, and showed that the CM-regularity of an algebra A can be considered as an invariant that measures how far away A is from being AS-regular for any Koszul noetherian connected graded algebra A with a balanced dualizing complex. In the last two years, Kirkman-Won-Zhang did a lot of work about the regularities, in particular they gave a far-reaching

generalization of Dong-Wu's result by introducing another numerical homological invariant ASreg(A) for any noetherian connected graded k-algebra A. Recently, Wu and Yi defined more numerical homological invariants over positively graded algebras and studied relations between them. I will concentrate to Wu-Yi's work in the talk.

Pseudoalgebras in a homomorphism category

Zhixiang Wu(吴志祥)
Zhejiang University

We mainly introduce an associative algebra in a new pseudotensor category based on a homomorphism category of left H-modules, where H is a Hopf algebra. This algebra is called an associative $\mathcal{H}H$-pseudoalgebra, which consists of an associative H-pseudoalgebra and a bipseudomodule over this associative H-pseudoalgebra. Basic properties, extensions of representations of an associative $\mathcal{H}H$-pseudoalgebra are described by a group and cohomology of associative $\mathcal{H}H$-pseudoalgebras is established.

Fusion Products of Twisted Modules in Pemutation Orbifolds

Nina Yu(余铌娜)
Xiamen University

Orbifold theory explores the structure of vertex operator algebras (VOAs) when finite groups act on them, with a particular focus on understanding the representation theory of the fixed point subalgebra. A key aspect of this field is permutation orbifolds, which study how the symmetric group acts on the n-fold tensor product of a VOA. In this talk, I will present our recent findings on the fusion product of twisted modules in permutation orbifolds. This work is a collaboration with C. Dong and F. Xu.

On the realization of a class of $SL(2,Z)$-representations

Zhiqiang Yu(于志强)
Yangzhou University

In this talk, I will talk about the realization of a class of irreducible representation of the modular group $SL(2,Z)$. Explicitly, let $p<q$ be odd primes, ρ_1 and ρ_2 be irreducible representations of $SL(2,Z/p)$ and $SL(2,Z/q)$ of dimensions $\frac{p+1}{2}$ and $\frac{q+1}{2}$, respectively. If $\rho_1\oplus\rho_2$ can be realized as modular representation associated to a modular fusion category $\mathcal{C}$, then $q-p=4$. Moreover, if $\mathcal{C}$ contains a non-trivial \'{e}tale algebra, then $\mathcal{C}$ is connected with a near-group fusion category of type $Z/p+p$, which gives a partial answer to the conjecture of D. Evans and T. Gannon. By using the Witt equivalence of fusion categories, I also introduce an infinite classes of potential realizable fusion rings which contain elements of Frobenius-Perron dimension $\frac{\sqrt{p}+\sqrt{q}}{2}$.

Modular Extension of Higher Fusion Categories

Hao Zheng(郑浩)

Tsinghua University

The concept of modular extension, first introduced by Muger, has proven to be intimately related to the symmetries of topological phases. In this talk, we provide a higher formulation of modular extension and present several classification results. Notably, we demonstrate the existence of a surprising long exact sequence formed by groups of modular extensions and quantum Witt groups.

Quantum Codes Using the τ-OD MP Construction

Kun Zhou(周坤)
BIMSA

We propose a method called the τ-optimal defining (τ-OD) matrix-product (MP) construction to derive infinite families of quantum codes with good parameters. Through this scheme, we present 100 record-breaking quantum codes, which exceed the best-known lower bounds on the minimum distances of quantum codes listed in Grassl's online database（Joint with Meng Cao）.

Yetter-Drinfeld modules over a quasi-triangular Hopf algebras, and its indecomposable module categories decomposition

Shenglin Zhu(朱胜林)
Fudan University

Let G be a finite group and $C = {}^{G}_{G}YD$ be the corresponding tensor category of YetterDrinfeld modules. Then the conjugacy class decomposition of G interprets C as a direct sum of indecomposable C-module categories, with each such indecomposable C-module category itself being a tensor category, which is isomorphic to a module category over a subgroup algebra of G. In this talk we extended this result to finite dimensional quasitriangular Hopf algebras, and try to find mechanism to reconstruct the tensor category ${}^{H}_{H}YD$ from the indecomposable module categories.